- The test contains 30 questions and you have 90 minutes to solve them.
- Each question has only one correct answer.
- For each correct answer, you get 1 point. For each incorrect answer, you lose 1/3 of a point. For each question you leave unanswered, you get 0 points.
- You may change your answers (the number of changes is not limited) or delete them.
- Unless indicated otherwise, all numbers used in the test are real numbers based on the decimal system.
- You may use formulas that are part of the test interface.
- You can zoom in or zoom out to make the text larger or smaller. In Google Chrome, you can find the zoom function by clicking on the three dots in the upper right corner of the browser window.

Question 1

What is the solution of the given equation?

$$\frac{12}{x} + \frac{8}{x} + \frac{4}{x} - \frac{20}{x} + 2 = 0$$

A $x = -4$

B $x = -2$

C $x = 0$

D $x = 2$

Question 2

What is the magnitude of the acute angle between the minute hand and the hour hand on a clock that shows half past seven?

A 30°

B 35°

C 42.5°

D 45°

Question 3

A number of the form $7k + 24$ is (for **any** integer value of k) **not divisible** by:

A 4

B 5

C 6

D 7

Question 4

Given three arbitrary sets A, B, C, the sets X, Y are defined by

$X = (A - B) \cap (A - C)$,

$Y = A - (B - C)$.

Then the following holds:

A $X = Y$

B $X \cap Y = \emptyset$

C $X \subseteq Y$

D $X \supseteq Y$

Question 5

The number of all positive integers that are divisible by 15 and are divisors of 50 equals:

A 0

B 1

C 2

D 3

Question 6

Among the given options, the smallest number is:

A $\frac{3\sqrt{2}}{\sqrt{3}}$

B $\frac{2\sqrt{3}}{\sqrt{6}}$

C $\frac{3\sqrt{3}}{2\sqrt{6}}$

D $\frac{2\sqrt{6}}{\sqrt{8}}$

Question 7

Which of the following statements holds?

A The number $\sqrt{2}$ has a finite decimal expansion.

B Any natural number is divisible by 8 if and only if its digit sum is divisible by 8.

C There exists at most one even prime number.

D For every negative number, its absolute value is equal to the reciprocal of the number.

Question 8

For every natural number n, the number

$5n^3 - 5n$

is divisible by:

A 20

B 24

C 30

D 36

Question 9

Peter proclaimed: "If I fail the test, I won't watch TV and I will study." Which of the following situations **is not** consistent with the statement?

A Peter failed the test, did not watch TV and did not study.

B Peter did not fail the test, watched TV and studied.

C Peter did not fail the test, did not watch TV and studied.

D Peter did not fail the test, watched TV and did not study.

Question 10

The equation $3x^2 - bx + 8 = 0$, where x is the unknown and b is a real parameter, has two distinct real roots whenever b is taken from the set:

A $(-\infty, -4\sqrt{6}) \cup (4\sqrt{6}, \infty)$

B $(\sqrt{3}, 4\sqrt{6})$

C $(-\infty, -3) \cup (3, \infty)$

D $(-\infty, \infty)$

Question 11

For what value of the real parameter p is the polynomial

$x^3 - 3x^2 + 2x + p$

divisible by the polynomial $x^2 + 2$?

A $p = -6$

B $p = 0$

C $p = 3$

D There is no p of such value.

Question 12

Consider the equation

$$4px^2 + (q - p) \cdot 3^x + 2 = 0,$$

where x is the unknown and p, q are parameters.

The values $x_0 = 0, x_1 = 1$ are solutions of the equation when the values of parameters p, q are:

A $p = q = -1$

B $p = 1, q = -1$

C $p = q = 1$

D $p = -1, q = 1$

Question 13

Which of the statements below is true about the equation $\frac{x^2 - 4}{2x - 4} = 2$?

A The equation does not have any real solution.

B The equation has exactly one positive solution.

C The equation has exactly one negative solution.

D The equation has exactly two distinct real solutions.

Question 14

How many numbers greater than 45,123 can be obtained by permuting the digits in the number 45,123?

A 11

B 23

C 29

D 47

Question 15

Denote by A the probability that 7 is rolled when rolling one twelve-sided die, and let B denote the probability that 7 is rolled as the sum of rolling a pair of six-sided dice.

Then $\frac{A}{B}$ equals:

A 1

B $\frac{1}{2}$

C $\frac{1}{3}$

D $\frac{1}{6}$

Question 16

Prior to a tennis tournament, six students of a tennis academy are divided into pairs for the first round of matches. There are two two eight-year-old students, three seven-year-old students and one six-year-old student. The organizers want to make sure that the six-year-old student is not paired with any of the eight-year old students in the first round. The number of pairings that satisfy this criterion is:

A 4

B 9

C 15

D 24

Question 17

Andrew's grades in mathematics are: 2, 3, 1, 2, 4, 1. The least number of 1's he additionally needs to get in order for his grade average to be less than 1.5 is:

A 7

B 8

C 9

D 11

Question 18

For what value of a does the following equality hold?

$\log_a 7 + \log_a 11 = 1$

A 2

B 77

C 1

D No value of a satisfies the equation.

Question 19

Which of the folowing is a maximal interval of non-negative values of the function $f: y = e^x - e^2$?

A $[e^2, \infty)$

B $[2, \infty)$

C $(-\infty, 2]$

D (e^2, ∞)

Question 20

The number of all solutions of the equation

$$\sin(\cos x) = 0$$

in the interval $[-\pi, \pi]$ equals:

A 0 (does not have a solution in the given interval)

B 1

C 2

D 4

Question 21

Given the first five terms of each of the following three sequences:

$$(a_n): 1, \frac{5}{3}, \frac{7}{3}, 3, \frac{11}{3}, \ldots,$$

$$(b_n): 0, \frac{5}{2}, \frac{7}{2}, 5, \frac{13}{2}, \ldots,$$

$$(c_n): -\frac{7}{5}, -1, -\frac{3}{5}, -\frac{1}{5}, \frac{1}{5}, \ldots.$$

Which of the sequences (a_n), (b_n), (c_n) **cannot** possibly be an arithmetic progression?

A only (a_n) and (b_n)

B only (b_n)

C only (c_n)

D Each of the sequences (a_n), (b_n), (c_n) may be an arithmetic progression.

Question 22

The codomain of the function $f: y = 1 - |x|$ **does not** contain the value:

A −2

B 0

C 1

D 2

Question 23

The overall number of points where the graph of the function

$$f: y = \frac{3}{x - 2} + 1$$

intersects any one of the lines $y = 0, x = 0, y = x$, is:

A 1

B 2

C 3

D 4

Question 24

The function $f: y = 2$, whose domain is $\mathbb{R}$, is identical with the function (whose domain thus also has to be $\mathbb{R}$):

A $a: y = 2x^2 \cdot x^{-2}$

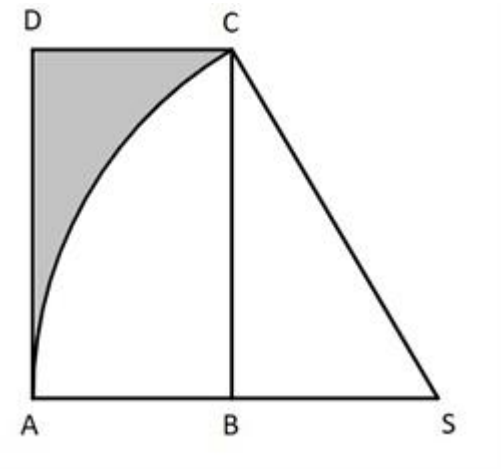
B $b: y = \dfrac{2x^2 - 10x + 12}{x^2 - 5x + 6}$

C $d: y = 2 \cdot \tan x \cdot \cot x$

D $e: y = \dfrac{2}{\sin^2 x + \cos^2 x}$

Question 25

Consider the figure below:



The points A, B, C, D are vertices of a rectangle whose side lengths are $|AB| = 1, |AD| = \sqrt{3}$. The vertices A and C are further connected by a circular arc whose center S lies on the ray AB. Moreover, the resulting triangle ASC is equilateral. The area of the grey region, enclosed by the line segments AD, DC and the circular arc, equals:

A $\frac{2\sqrt{3}}{3} - \pi$

B $\frac{3\sqrt{3}}{2} - \frac{2}{3}\pi$

C $\frac{\sqrt{3}}{2} - \frac{2}{3}\pi$

D $3\sqrt{3} + \pi$

Question 26

A triangle has side lengths $\frac{1}{2}a$, a, $a\sqrt{2}$, where a is a positive real number. The cosine of the largest vertex angle equals:

A $-\frac{3}{4}$

B 0

C $\frac{2}{3}$

D $\frac{3}{4}$

Question 27

Consider a triangle *ABC* in the plane with the following properties:

The midpoint of the side *AB* is (2, 0),

the midpoint of the side *BC* is (4, 2),

the midpoint of the side *AC* is (3, -1).

Then the vertex *C* is:

A (-2, 3)

B (5, 1)

C (6, 2)

D (4, 0)

Question 28

If the point $O(0, 0)$ in the plane is translated by the vector $v = (4, 2)$, the resulting point O' is the reflected image of O across a line p. Then an equation of the line p is:

A $2x + 4y - 3 = 0$

B $2x + y - 5 = 0$

C $x + y = 0$

D $2x + y - 3 = 0$

Question 29

A metal ball is thrown into a glass of cylindrical shape filled with water. The ball sinks completely, causing the water level to rise by 1 cm. Given that the diameter of the ball is 6 cm, the inner diameter of the glass is:

A 8 cm

B 12 cm

C $\frac{7}{3}\pi$ cm

D $6\sqrt{\pi}$ cm

Question 30

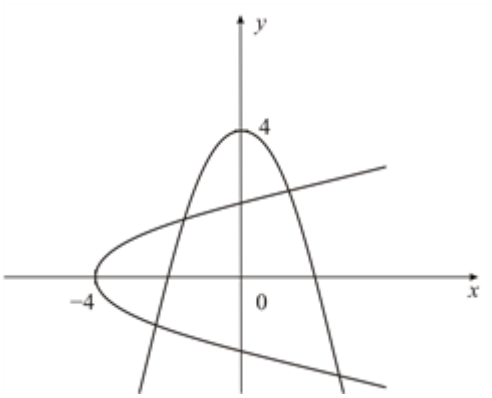
Two parabolas are given by the equations

$x^2 - y = -4$,

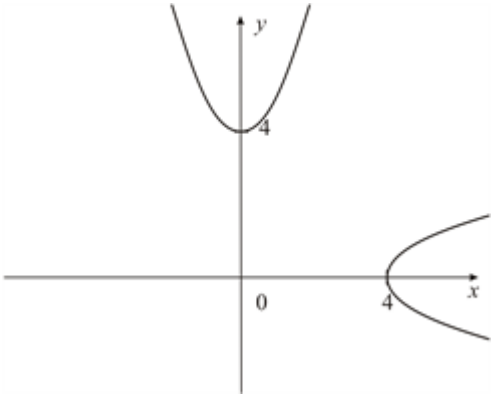
$x + y^2 = 4$.

Which of the figures below shows this pair of parabolas?

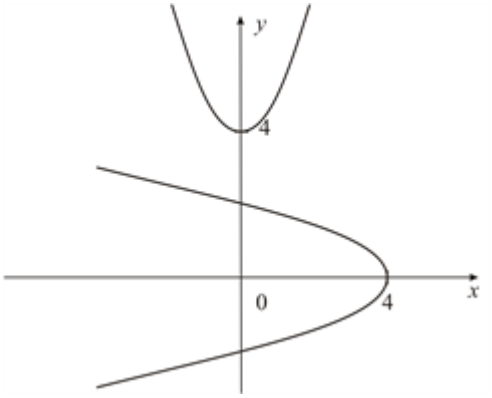
A



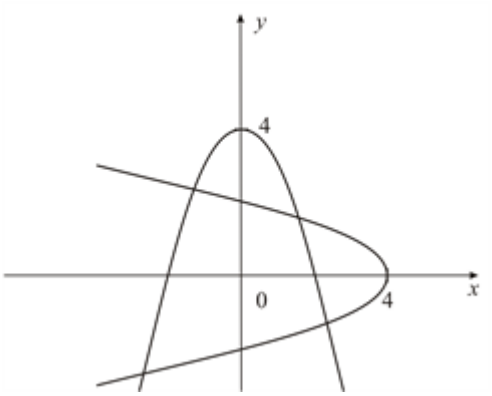
B



C



D



Correct solutions

1-B, 2-D, 3-D, 4-C, 5-A, 6-C, 7-C, 8-C, 9-A, 10-A, 11-A, 12-B, 13-A, 14-C, 15-B, 16-B, 17-C, 18-B, 19-B, 20-C, 21-B, 22-D, 23-D, 24-D, 25-B, 26-A, 27-B, 28-B, 29-B, 30-C